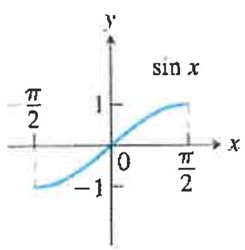
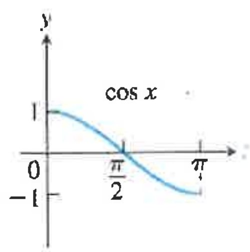
Inverse Trigonometric Functions

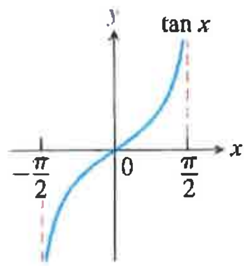
We restrict the domains of the six trigonometric functions so that they will be 1-1 and therefore have inverses.



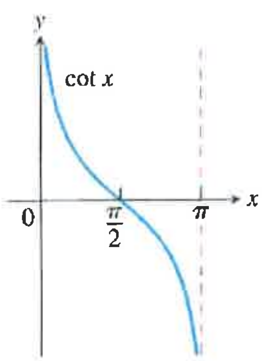
$y = \sin x$
Domain: $[-\pi/2, \pi/2]$
Range: $[-1, 1]$



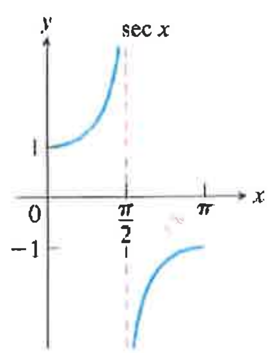
$y = \cos x$
Domain: $[0, \pi]$
Range: $[-1, 1]$



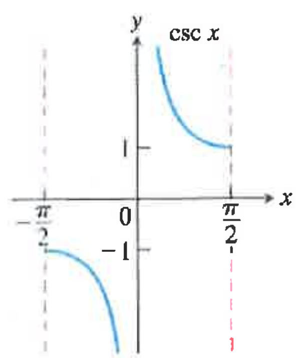
$y = \tan x$
Domain: $(-\pi/2, \pi/2)$
Range: $(-\infty, \infty)$



$y = \cot x$
Domain: $(0, \pi)$
Range: $(-\infty, \infty)$



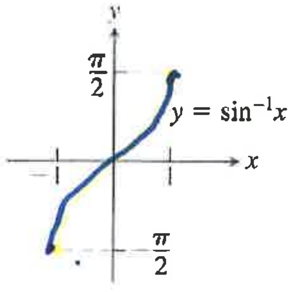
$y = \sec x$
Domain: $[0, \pi/2) \cup (\pi/2, \pi]$
Range: $(-\infty, -1] \cup [1, \infty)$



$y = \csc x$
Domain: $[-\pi/2, 0) \cup (0, \pi/2]$
Range: $(-\infty, -1] \cup [1, \infty)$

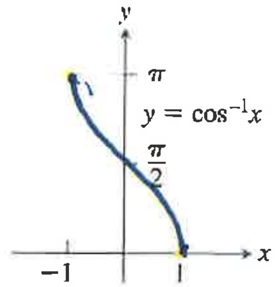
$\sin^{-1} x$, $\arcsin x$, $a\sin x$

Domain: $-1 \le x \le 1$
Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



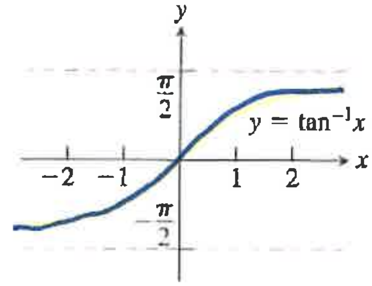
(a)

Domain: $-1 \le x \le 1$
Range: $0 \le y \le \pi$



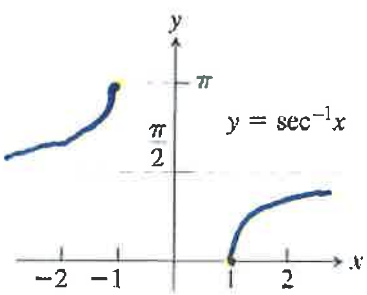
(b)

Domain: $-\infty < x < \infty$
Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

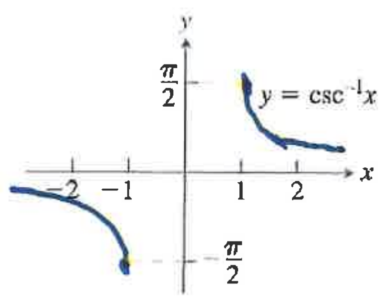


(c)

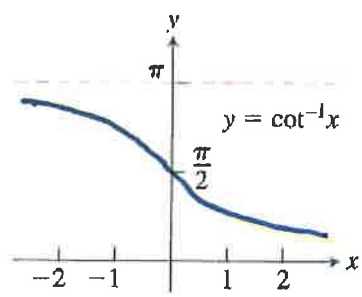
Domain: $x \le -1$ or $x \ge 1$
Range: $0 \le y \le \pi, y \ne \frac{\pi}{2}$



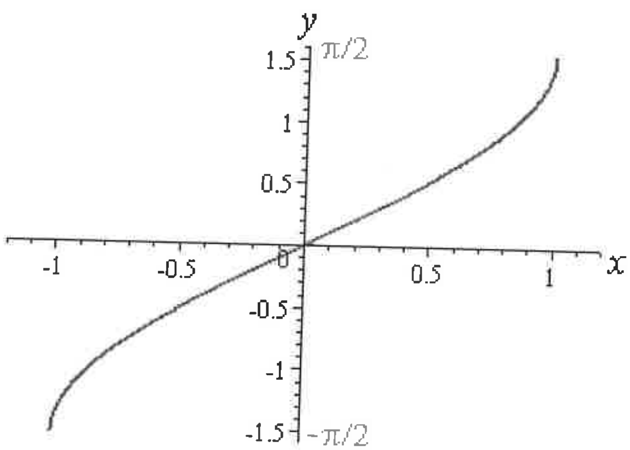
Domain: $x \le -1$ or $x \ge 1$
Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}, y \ne 0$



Domain: $-\infty < x < \infty$
Range: $0 < y < \pi$



$g(x) = sin^{-1}\, x$



Domain: $-1 \leq x \leq 1$; Range: $-\frac{\pi}{2} \leq sin^{-1}\, x \leq \frac{\pi}{2}$

Simplify the following:

$sin^{-1}\, \frac{1}{2}$

$\frac{\pi}{6}$ or $30°$

$sin^{-1} - \frac{\sqrt{3}}{2}$

$-\frac{\pi}{6}$ or $-60°$

$sin^{-1}\, 0$

0

$sin^{-1}\, 2$

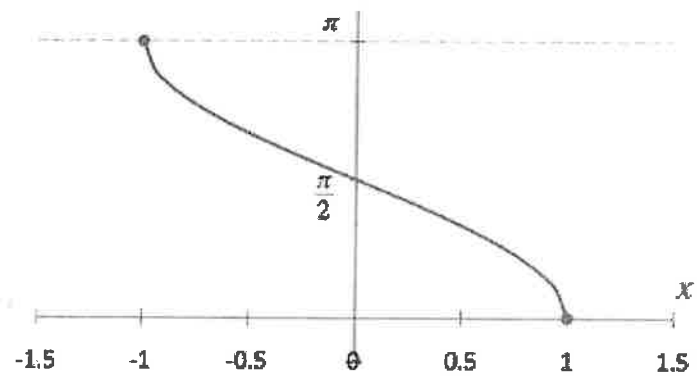
Not defined

$sin^{-1} - \frac{2}{3}$

$= -.7297$

calculator

$g(x) = cos^{-1} x$



Domain: $-1 \leq x \leq 1$; Range: $0 \leq cos^{-1} x \leq \pi$

Simplify the following:

$cos^{-1} \frac{1}{2}$

$\frac{\pi}{3}$ or $60°$

$cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\frac{5\pi}{6}$ or $150°$

$cos^{-1} 0$

$\frac{\pi}{2}$ or $90°$

$cos^{-1} 2$

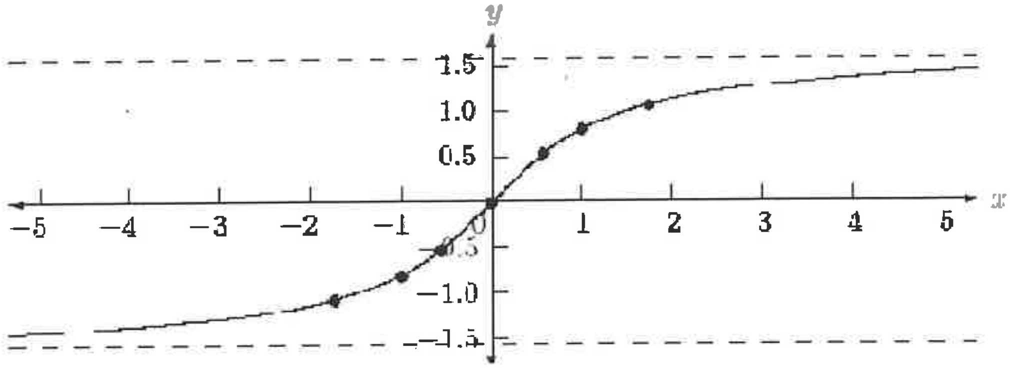
Not defined

$cos^{-1} -\frac{2}{3}$

$= 2.3005$

calculator

$f(x) = \tan^{-1} x$



Domain: $-\infty \leq x \leq \infty$; Range: $-\frac{\pi}{2} < tan^{-1} x < \frac{\pi}{2}$

Simplify the following:

$tan^{-1} 1$

$\frac{\pi}{4}$ or 45°

$tan^{-1} (-\sqrt{3})$

$-\frac{\pi}{3}$ or -60°

-1.0472

$tan^{-1} 0$

0

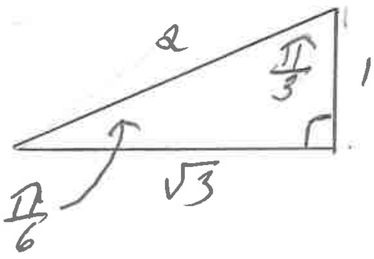
$tan^{-1} 2$

$= 1.1071$

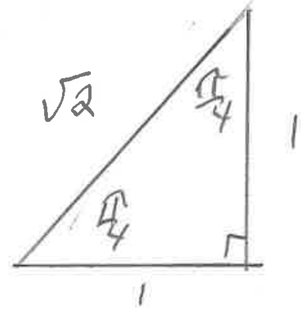
↑ calculator

Evaluate the following in exact form (In radians):

$sec^{-1}(2) = \frac{\pi}{3}$



$csc^{-1}\left(-\sqrt{2}\right) = -\frac{\pi}{4}$



$csc^{-1}(1)$

$\frac{\pi}{2}$

$sec^{-1}(\frac{1}{2})$

Not defined

Check your understanding:

What is $sin^{-1}(\frac{1}{3})$

$= .3398$ radians

Simplify the following expressions:

$sin(sin^{-1} \frac{\sqrt{3}}{2}) =$ $\frac{\sqrt{3}}{2}$

$cos(cos^{-1} 1) =$ 1

$f[f^{-1}(x)] = x$

$sin\,(sin^{-1} 0.37) =$ $.37$

$cos(cos^{-1} \frac{3}{2}) =$

Not defined

$tan\,(tan^{-1} 1) =$ 1

$tan(tan^{-1} \frac{3}{2}) =$ $\frac{3}{2}$

Combining Trig Functions and Inverse Trig Functions:

Evaluate the following expressions:

$sin\,(sin^{-1}x) = x$ for all x such that $-1 \leq x \leq 1$

$cos\,(cos^{-1}x) = x$ for all x such that $-1 \leq x \leq 1$

$tan\,(tan^{-1}x) = x$ for all x

Simplify the following expressions:

$sin^{-1}\left(sin\ \frac{\pi}{4}\right) =$

$\frac{\pi}{4}$

$sin^{-1}\left(sin\ \frac{7\pi}{4}\right) =$

$-\frac{\pi}{4}$

$sin^{-1}\left(sin\ \frac{6\pi}{5}\right) =$

$\sin^{-1}(-.58778)$

$= -.6283$

$cos^{-1}\left(cos\ \frac{3\pi}{8}\right) =$

$\cos^{-1}(.38268)$

$= 1.17809$

$cos^{-1}\left(cos\ \frac{9\pi}{8}\right) =$

$\cos^{-1}(-.92387)$

$= 2.74889$

$tan^{-1}\left(tan\ \pi\right) =$

$\tan^{-1}(0) = 0$

Combining Trig Functions and Inverse Trig Functions:
Evaluate the following expressions:

$sin^{-1}\left(sin\ x\right) = x$ for all x such that $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

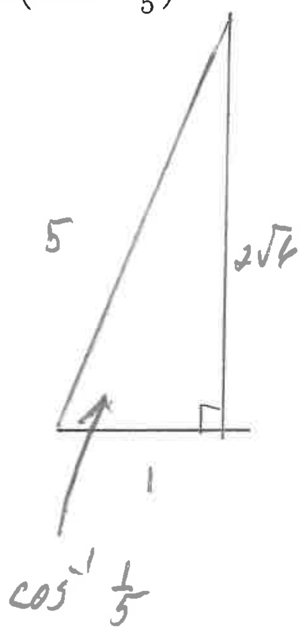
$cos^{-1}\left(cos\ x\right) = x$ for all x such that $0 \le x \le \pi$

$tan^{-1}\left(tan\ x\right) = x$ for all x such that $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Using Triangles to evaluate combinations of trig functions and inverse trig functions:

Remember that an inverse trig expression can be interpreted as an angle with given trigonometric properties. For instance, the expression $sin^{-1}x$ is an angle whose sin is x, the expression $cos^{-1}x^2$ is an angle whose cos is x^2. Use this idea along with diagrams involving right triangles to evaluate the trigonometric expressions below.

$tan(cos^{-1}\frac{1}{5})$



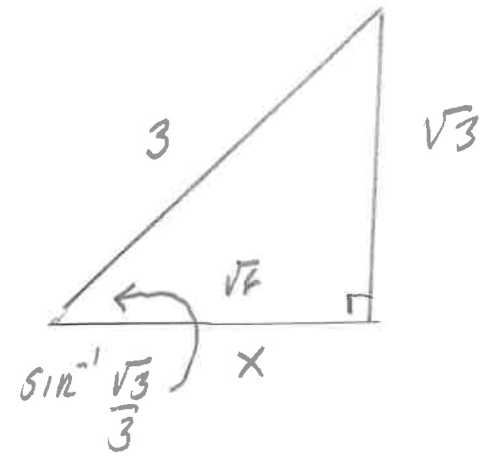
$25 = 1 + x^2$

$x^2 = 24$

$x = \sqrt{24}$

$x = 2\sqrt{6}$

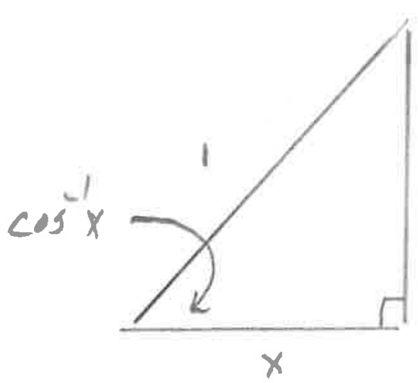
$sec(sin^{-1}\frac{\sqrt{3}}{3}) = \frac{3}{\sqrt{6}}$



$9 = x^2 + 3, \quad x^2 = 6$

$x = \sqrt{6}$

$sin\,(cos^{-1}x)$



$tan(sin^{-1}(x^2)) = \frac{x^2}{\sqrt{1-x^2}}$